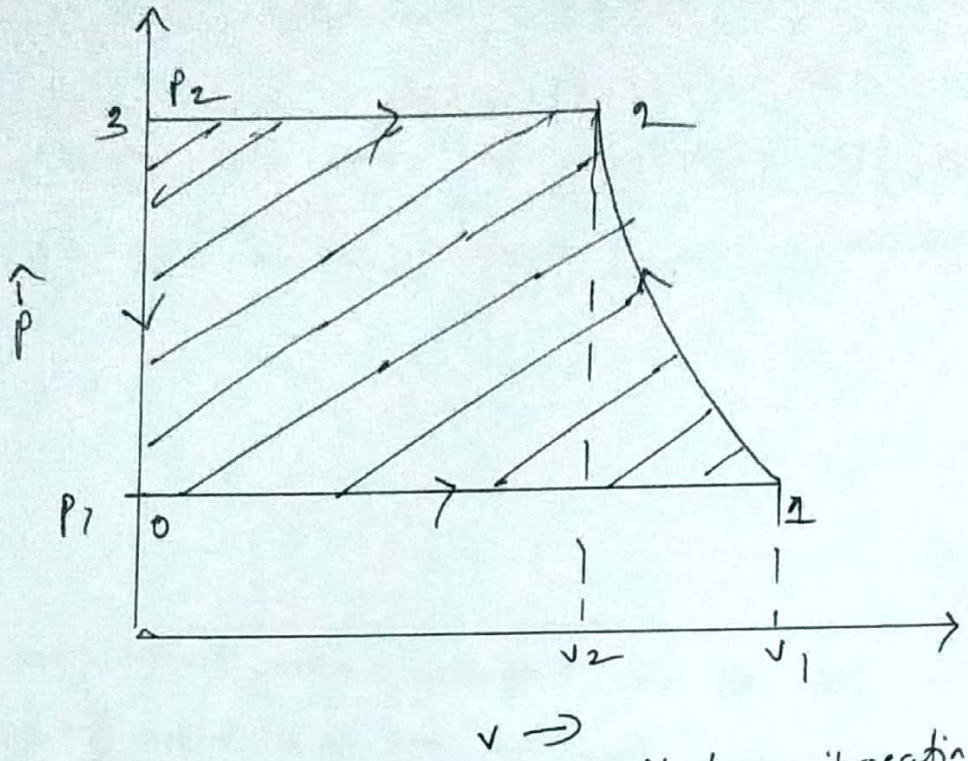
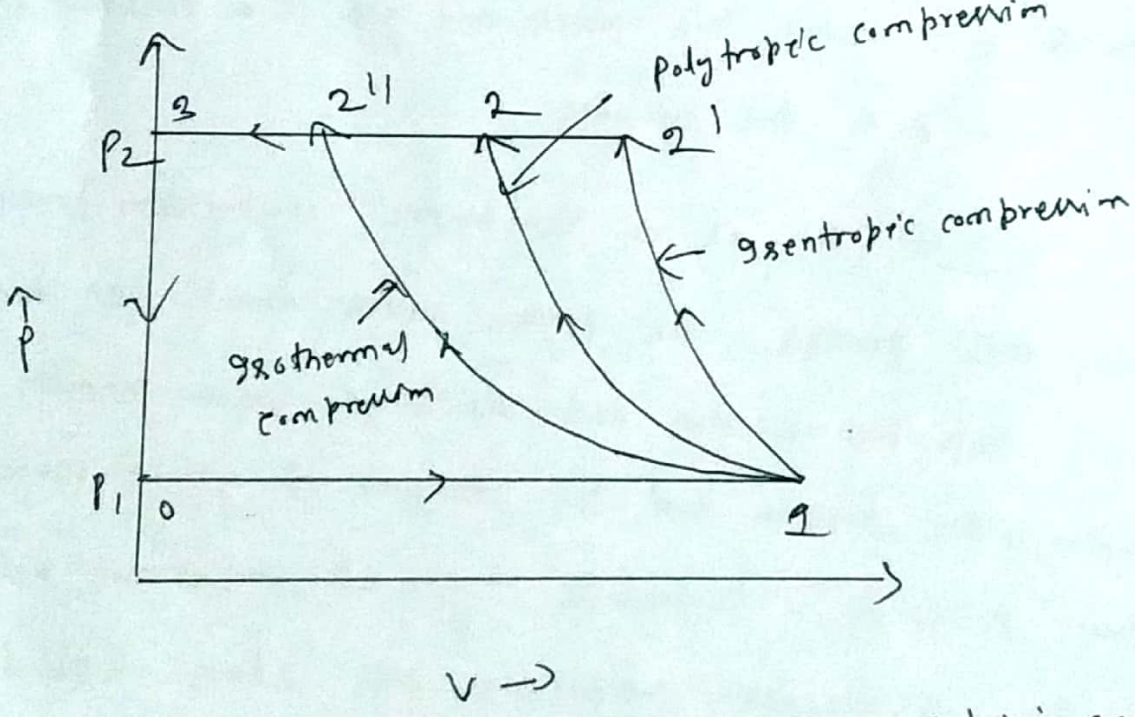
Q11 Derive the expression for work done by air compressor. [BPUT 2nd sem 2018-19]

Process 0-1 :- This is an isobaric suction process, during which the piston moves from the top dead centre (TDC) to the bottom dead centre (BDC). The suction valve remains open during this process and gas at a constant pressure. P_1 flows in to the cylinder.

Process 1-2 :- This is an isentropic compression process. During this process, the piston moves from BDC towards TDC. Both the suction and discharge valves remain closed during the process and the pressure of gas increases from P_1 to P_2. Various types of compression process in the reciprocating compressors are shown in the figure (b)



< P-V diagram for ideal reciprocating compressor >



< P-V diagram for isentropic, polytropic and isothermal compression >

Process 2-3 :- This is an isobaric discharge process. During this process, the suction valve remains closed and the discharge valve opens. Gas at a constant P_2 is expelled from the compressor as the piston moves to TDC.

Polytropic Compression

Area 0-1-2-3 in figure - (a) represents the net work done when the compression follows the polytropic law.

Net work on air per cycle = Area 0-1-2-3

= Work done during compression (1-2) + Work done during air delivery (2-3) — Work done during suction (0-1)

$$= \frac{P_2V_2 - P_1V_1}{n-1} + P_2V_2 - P_1V_1 = \frac{n}{n-1}(P_2V_2 - P_1V_1)$$

$$= \frac{n}{n-1} P_1V_1 \left(\frac{T_2}{T_1} - 1\right) = \frac{n}{n-1} P_1V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W = \frac{nm}{n-1} RT_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

Similarly, for isentropic compression

$$W = \frac{\gamma m}{\gamma - 1} RT_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

Isothermal Compression

$$W = \text{area } 0-1-2''-3$$

$$= P_2 V_2'' + P_1 V_1 \log_e \frac{V_1}{V_2''} - P_1 V_1$$

$$\langle P_2 V_2'' = P_1 V_1 \rangle$$

$$\Rightarrow W = P_1 V_1 \log_e \frac{V_1}{V_2''} = P_1 V_1 \log_e \frac{P_2''}{P_1}$$

* In isothermal compression work done is minimum and it is maximum for adiabatic compression.